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Th. 2. A subset of a countable set is countable. (4)

Proof:- Let A be a countable set.

Finite | Infinite

↓ | ↓

If then, we can have $B \subset A$ | If B is infinite

$\Rightarrow$ Since B is finite | ↓

$\Rightarrow$ B is ~~der~~ countable | Elements of B will be among the elements of A

But elements of A are in the form of sequen[ce]
$\Rightarrow$ elements of B can be considered / taken as $B = \{a_1, a_2, a_3, a_4 \ldots\}$

choose n_1 be the least positive integer st. $a_{n_1} \in B$.

Again let n_2 be the next least integer greater than n_1 st. $a_{n_2} \in B$. - - -

In this way, we can arrange the elements of B like that

$$B = \{a_{n_1}, a_{n_2}, a_{n_3}, \ldots\}$$

$f : N \to B$ st. $f(k) = a_{n_k} \; \forall k \in N$

$\Rightarrow B \sim N. \Rightarrow$ Denumerable

$\Rightarrow$ B is countable

Th. The union of a finite set and a Countable set is countable.

Proof:- Let A be a finite set.
Again let B be a countable set.
To prove that $A \cup B$ is countable.

Case(i) Let B is finite
$\Rightarrow A \cup B$ is again finite
$\Rightarrow A \cup B$ is countable. H.P.-I

Case(ii) Let B is infinite

Subcase (a): If $A \cap B$ is ϕ
$\Rightarrow$ No element is common between A and B.

Let $A = \{a_1, a_2, \ldots a_m\}$ $\because$ A is finite.
Let $B = \{b_1, b_2, b_3, \ldots\}$
$A \cup B = \{a_1, a_2, a_3, \ldots a_m; b_1, b_2, b_3, \ldots\}$.

Now; we a mapping f, from N to $A \cup B$ as follows.

$$f : N \rightarrow A \cup B$$

$$\text{s.t.} \quad f(n) = \begin{cases} a_n & ; \ 1 \le n \le m \\ b_{n-m} & ; \ m < n \end{cases}$$

This is a one-one-onto mapping
$\Rightarrow N \sim A \cup B$
$\Rightarrow A \cup B$ is countable. —— H.P-II

Subcase (b):- If some elements of A and B are common.
i.e. $A \cap B \neq \phi$